

Chapter 7

4 marks

SETS , RELATIONS AND FUNCTIONS

TOPIC :

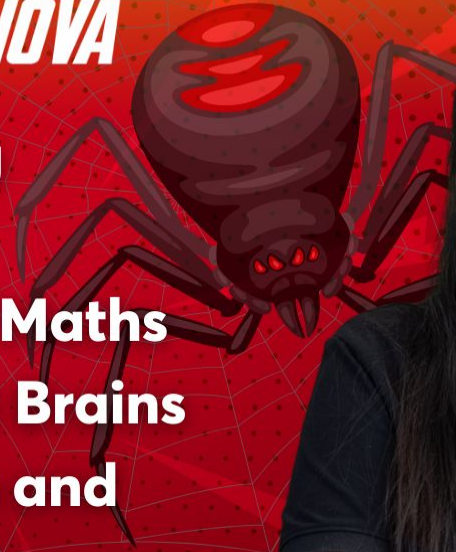
- SETS
- RELATIONS
- FUNCTIONS
- LIMITS AND CONTINUITY

BY : SHIVANI SHARMA

SHIVANI SHARMA

QUANTITATIVE APTITUDE ROMANOVA

- **7 years** of experience in teaching **Mathematics**
- **Gold Medalist** in M.Sc. and B.Sc. Maths
- Was **#1 Maths Faculty** in Magnet Brains
- Teaches Maths in **CA foundation** and **K-12**
- Taught **30000+ students** with highest score as 100



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Plus Learner

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Learners
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Learners scored
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Top Scorer

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Jain**

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Jain**

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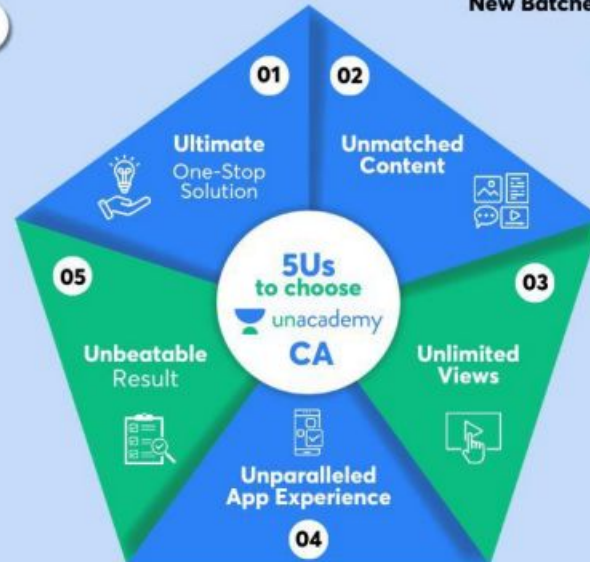
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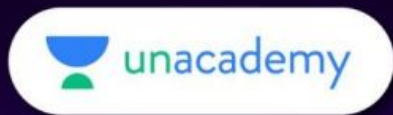
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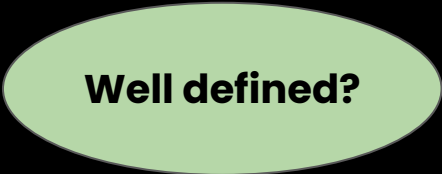
**COLLECTION OF ALL THE
DAYS OF A WEEK**



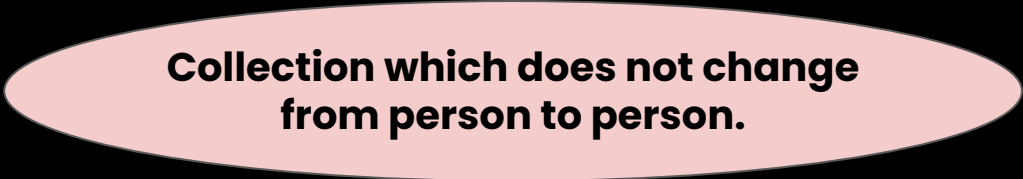
SETS



A well-defined collection of objects is called a set.



Well defined?



Collection which does not change from person to person.

$$A = \{a, e, i, o, u\}$$

- We denote sets by capital letters A, B, C, X, Y, Z , etc.
- The objects in a set are called its members or elements.
- Elements are usually denoted by small letters.
- If a is an element of a set A , we write, $a \in A$, which means that a belongs to A or that a is an element of A .

POINT TO REMEMBER

i. The elements of a set may be listed in any order

Thus, $\{1, 2, 3\} = \{2, 1, 3\} = \{3, 2, 1\}$.

ii. The repetition of elements in a set has no meaning.

Thus, $\{1, 2, 3\} = \{1, 1, 2, 3, 2\} = \{1, 1, 2, 2, 3, 3, 3\}$, etc.

CARDINALITY / CARDINAL NUMBER

The number of distinct elements in a set is called as Cardinal number of it .

For a set A , we represent cardinality with $n(A)$.

Example:

Let $A = \{1, 3, 5\}$

Then,

$$n(A) = 3$$

PRESENTATION OF A SET

Descriptive Form

D = The set of odd digits between 1 and 9 both inclusive.

**Roster/Tabular Form/
Braces Form**

$D = \{1, 3, 5, 7, 9\}$

**Set Builder Form/Algebraic
Form/Rule Method/
Property Method**

$D = \{x : x = 2n - 1, \text{ where } n \in \mathbf{N} \text{ and } 1 \leq n \leq 5\}$

Roster/Tabular Form

- In the roster form, we list all the members of the set within braces $\{ \}$ and separate them by commas.

Set builder Form

- In the set-builder form, we list the property or properties satisfied by all the elements of the set.

- We write,

$\{x : x \text{ satisfies properties } P\}$, which is read as 'the set of all

those x such that each x has properties P '.

EXERCISE 7 (A)

Que 5. The set $\{x \mid 0 < x < 5\}$ represents the set when x may take integral values only

- a. $\{0, 1, 2, 3, 4, 5\}$
- b. $\{1, 2, 3, 4\}$
- c. $\{1, 2, 3, 4, 5\}$
- d. None of these

ANS : b

EXERCISE 7 (A)

Que 6. The set $\{0, 2, 4, 6, 8, 10\}$ can be written as

- a. $\{2x \mid 0 < x < 5\}$
- b. $\{x : 0 < x < 5\}$
- c. $\{2x : 0 \leq x \leq 5\}$
- d. None of these

ANS : C

TYPES OF SETS

Empty Set:

- A set containing no element at all is called the empty set or the null set or the void set, denoted by ϕ . or $\{ \}$.

Example:

i. $\{x : x \in \mathbb{N} \text{ and } 2 < x < 3\} = \phi$.

TYPES OF SETS

Singleton Set:

- A set containing exactly one element is called a singleton set.

Example:

i. $\{x : x \in \mathbb{Z} \text{ and } x + 4 = 0\} = \{-4\}$, which is a singleton set.

TYPES OF SETS

Finite sets

- An empty set or a non-empty set in which the process of counting of elements surely comes to an end is called a finite set.
- The number of distinct elements contained in a finite set A is denoted by $n(A)$.

Example:

i. Let $A = \{ 2, 4, 6, 8, 10, 12 \}$.

Then, A is clearly a finite set and $n(A) = 6$.

TYPES OF SETS

Infinite Sets:

- A set which is not finite is called an infinite set .

EXAMPLE

- ii. $\mathbb{N}$: the set of all natural numbers .
- iii. $\mathbb{Z}$: the set of all integers.

Jan 2021

The set of cubes of natural number is

- (a) Null set**
- (b) A finite set**
- (c) An infinite set**
- (d) Singleton Set**

ANS : C

USE MY CODE : SS12

TYPES OF SETS

Equal Set:

- Two non-empty sets A and B are said to be equal, if they have exactly the same elements and we write, $A = B$.

EXAMPLE :

Let A = Set of letters in the word 'follow'

B = Set of letters in the word 'wolf '

Here ,

$$A = B$$

TYPES OF SETS

Equivalent Set:

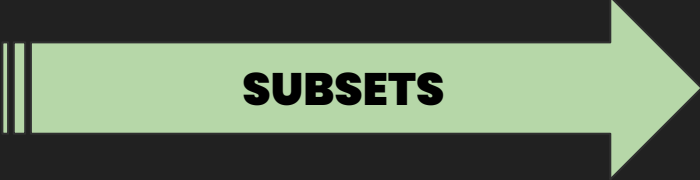
- Two finite sets A and B are said to be equivalent, if $n(A) = n(B)$.
- Equal sets are always equivalent. But, equivalent sets need not be equal.

Example:

i. Let $A = \{1, 3, 5\}$ and $B = \{2, 4, 6\}$.

Then, $n(A) = n(B) = 3$.

So, A and B are equivalent.



SUBSETS

A set A is called a subset of a set B if every element of A is also element of B



SUPERSET

If 'A' is subset of 'B' then 'B' is called a Superset of 'A'



PROPER SUBSET

When A is a subset of B but A is not equal to B , then A is a proper subset of B .

$$A \subset B$$



IMPROPER SUBSET

If A is a subset of B and also B is a subset of A , then both are improper subsets of each other , this is possible only when both the sets are equal.

$$A \subseteq B$$



FORMULAE

- No. of **possible subsets** of set containing **n elements**

$$2^n$$

- No. of **proper subsets** of set containing **n elements**

$$2^n - 1$$



FORMULAE

- **Every set is a subset of itself.**
- **The empty set is a subset of every set .**



POWER SET

- **Let A be a set . Then the collection of all subsets of A is called the power set of A and is denoted by $P(A)$**
- **A is a finite set having n elements , then $P (A)$ has 2^n elements**

JUNE 2012 , MAY 2018

The numbers of proper subsets of the set $\{ 3, 4, 5, 6, 7 \}$ is:

(a) 32

(b) 31

(c) 30

(d) 25

ANS: b

USE MY CODE : SS12

JUNE 2022 , DEC 2020

Two finite sets have m and n elements. The total number of sub – sets of the first set is 56 more than the total number of sub –sets of the second set. The values of m and n are

(a) 6 , 3

(b) 7 , 6

(c) 5 , 1

(d) 8 , 7

ANS : a

USE MY CODE : SS12

UNIVERSAL SET

Example:

- Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$ and $C = \{6, 7\}$.

If we consider the set $U = \{1, 2, 3, 4, 5, 6, 7\}$, then clearly, U is a superset of each of the given sets.

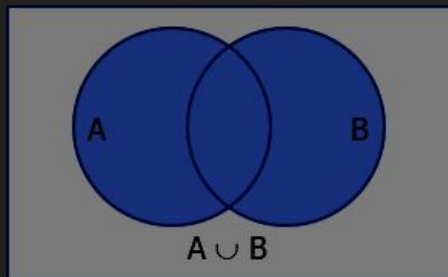
Hence, U is the universal set.

If there are some sets under consideration, then there happens to be a set which is a superset of each one of the given sets. Such a set is known as the universal set for those sets. We shall denote a universal set by U .

OPERATIONS ON SETS

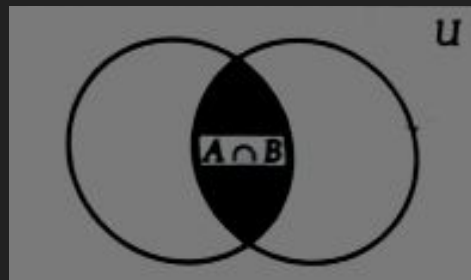
UNION

$$A \cup B = \{x : x \in A \text{ or } x \in B\}.$$



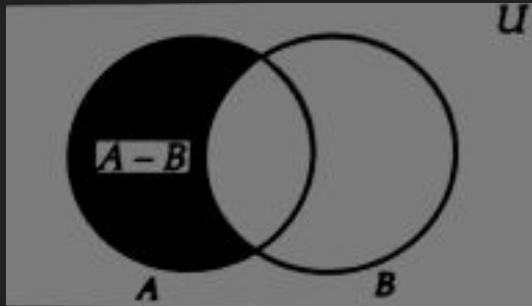
INTERSECTION

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

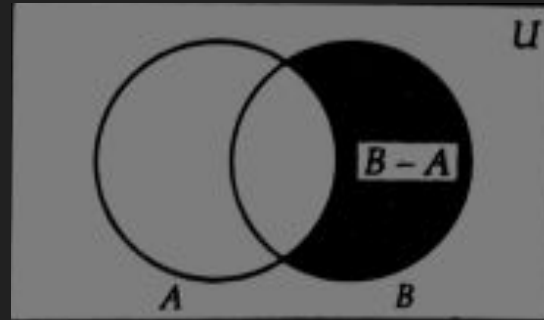


DIFFERENCE OF SETS

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

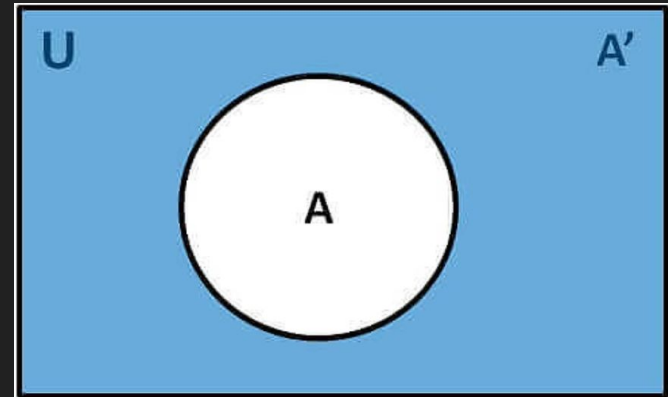


$$B - A = \{x \in B \text{ and } x \notin A\}$$



COMPLEMENT OF A SET

- Let U be the universal set and let A be a set such that $A \subset U$. Then, the complement of A with respect to U is denoted by A' or A^c or $U - A$ and is defined the set of all those elements of U which are not in A



JUNE 2019

If $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$; $B = \{1, 3, 4, 5, 7, 8\}$; $C = \{2, 6, 8\}$

Then find $(A-B) \cup C$

(a) $\{2, 6\}$

(b) $\{2, 6, 8\}$

(c) $\{2, 6, 8, 9\}$

(d) None

ANS: C

USE MY CODE : SS12

JUNE 2019

If $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{2, 4, 6, 8\}$. Cardinal number of $A - B$ is:

(a) 4

(b) 3

(c) 9

(d) 7

Ans: a

USE MY CODE : SS12



DE MORGAN'S LAW

For any two sets A and B,

(i) $(A \cup B)' = (A' \cap B')$

(ii) $(A \cap B)' = (A' \cup B')$

May 2018

Let U be the universal set, A and B are the subsets of U . If $n(U)=650$, $n(A) = 310$, $n(A \cap B) = 95$ and $n(B) = 190$. then $n(\bar{A} \cap \bar{B})$ is equal to

(a) 400

(b) 200

(c) 300

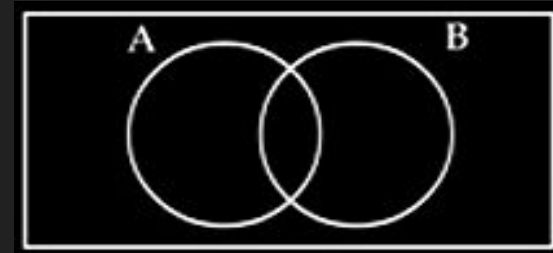
(d) 245

Ans:d

USE MY CODE : SS12

FORMULAE

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$



- $n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(P \cap R) + n(P \cap Q \cap R)$



May 2018

In a town of 20,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C. 5% families buy A and B, 3% buy B and C and 4% buy A and C if 2% families buy all the three newspapers, then the number of families which buy A only is:

(a) 6600

(b) 6300

(c) 5600

(d) 600

Ans : a

USE MY CODE : SS12

Ordered Pair

- Two numbers a and b listed in a specific order and enclosed in parentheses form an ordered pair (a, b) .

$$(a, b) \neq (b, a)$$

Cartesian Product of Two Sets

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}.$$



- Let A and B be two sets. Then a relation R from set A to set B is a subset of $A \times B$.
- Thus, R is a relation from A to $B \Leftrightarrow R \subseteq A \times B$
- If A and B are finite sets consisting of m and n elements respectively then $A \times B$ has

mn

total number of relations from A to B is 2^{mn} .

Nov 2018

If $A = \{1, 2\}$ and $B = \{3, 4\}$, Determine the number of relations from A and B:

(a) 3

(b) 16

(c) 5

(d) 6

Ans : b

USE MY CODE : SS12

DOMAIN , RANGE , CODOMAIN OF A RELATION

➤ If $A = \{1, 3, 5, 7\}$

$B = \{2, 4, 6, 8, 10\}$ and R is relation from A to B

$$R = \{(1, 8), (3, 6), (5, 2), (1, 4)\}$$

June 2014

The range of the relation $\{(1, 0) (2, 0) (3, 0) (4, 0) (0, 0)\}$ is

(a) $\{1, 2, 3, 4, 0\}$

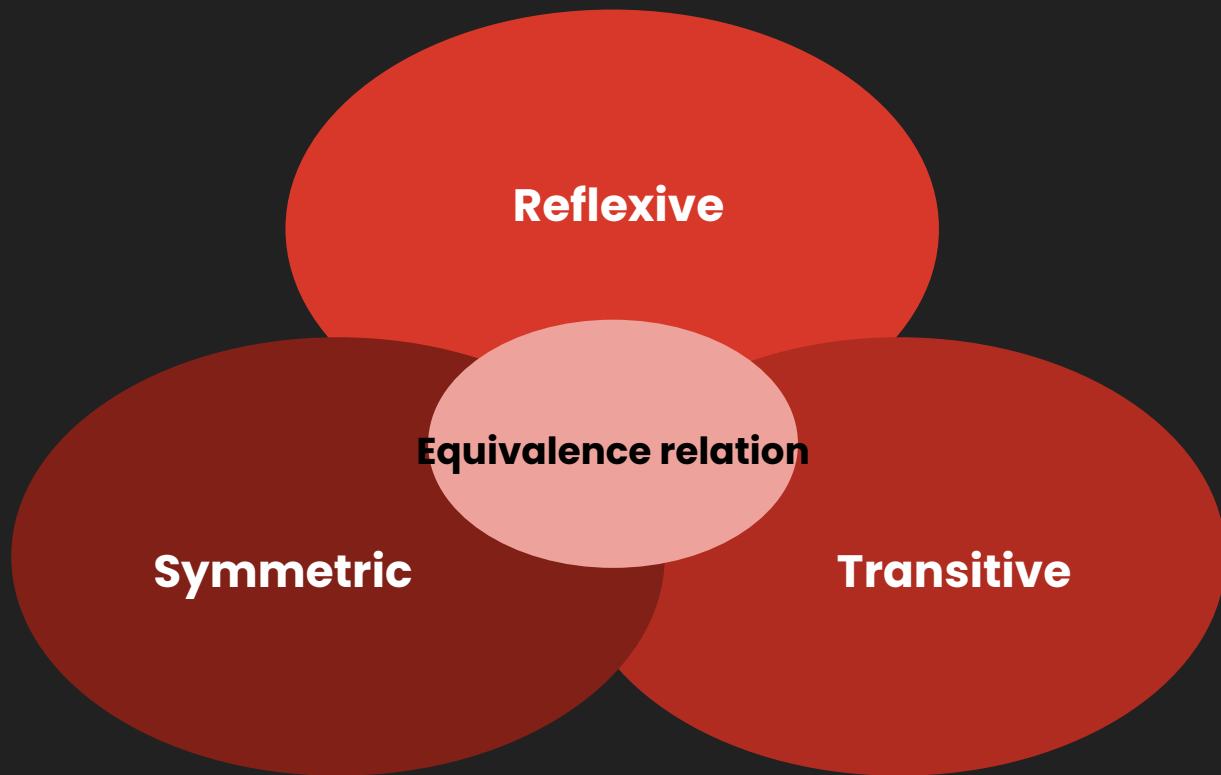
(b) $\{0\}$

(c) $\{1, 2, 3, 4\}$

(d) None

Ans : b

USE MY CODE : SS12



REFLEXIVE RELATION

Example: $A = \{1, 2, 3\}$ and R_1, R_2, R_3 be the relations given as

$$R_1 = \{(1, 1), (2, 2), (3, 3)\}$$

Reflexive relation

$$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3)\}$$

Reflexive relation

$$R_3 = \{(2, 2), (2, 3), (3, 2), (1, 1)\}$$

Not Reflexive relation

SYMMETRIC RELATION

Example: Let $A = \{1, 2, 3\}$ and R_1, R_2, R_3 be the relations on A

$$R_1 = \{(1, 2), (2, 1)\},$$

Symmetric relation

$$R_2 = \{(1, 2), (2, 1), (1, 3), (3, 1)\}$$

Symmetric relation

$$R_3 = \{(1, 3), (3, 1), (2, 3)\}$$

Not Symmetric relation

TRANSITIVE RELATION

Example: Let $A = \{1, 2, 3\}$ and R_1 and R_2 in A be defined as

$$R_1 = \{(1, 2), (2, 3), (1, 3), (3, 2)\}$$

Not Transitive relation

$$R_2 = \{(1, 3), (3, 2), (1, 2)\}$$

Transitive relation

Dec 2014

If a relation $S = \{ (1,1), (2,2), (1,2), (2,1) \}$ on $S = \{ 1,2,3 \}$ is symmetric and

- (a) Reflexive but not transitive**
- (b) Reflexive as well as transitive**
- (c) Transitive but not reflexive**
- (d) Neither transitive nor reflexive**

Ans : c

USE MY CODE : SS12

Jan 2021

In the set of all straight lines on a plane which of the following is Not 'TRUE'?

- (a) Parallel to an equivalence relation**
- (b) Perpendicular to is a symmetric relation**
- (c) Perpendicular to is an equivalence relation**
- (d) Parallel to a reflexive relation**

Ans : c

USE MY CODE : SS12

IDENTITY RELATION

If $A = \{1, 2, 3\}$

Let R_1, R_2, R_3 be relation on A

$$R_1 = \{(1, 1), (2, 2), (3, 3)\}$$

Identity relation

$$R_2 = \{(1, 1), (2, 2)\}$$

Not Identity relation

$$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$$

Not Identity relation

INVERSE RELATION

$$R^{-1} = \{ (b, a) : (a, b) \in R \}$$

EXAMPLE

Let $A = \{ 1, 2, 3 \}$,

$B = \{ a, b, c, d \}$

be two sets and

let $R = \{ (1, a), (1, c), (2, d), (2, c) \}$ be a relation from
 A to B .

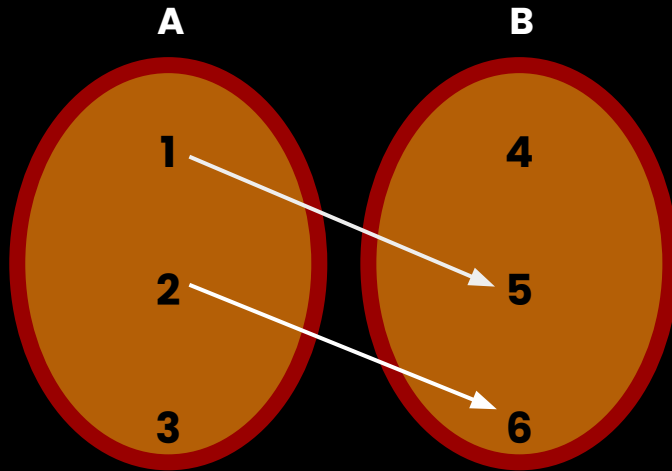
Then, $R^{-1} = \{ (a, 1), (c, 1), (d, 2), (c, 2) \}$ is a relation
from B to A .



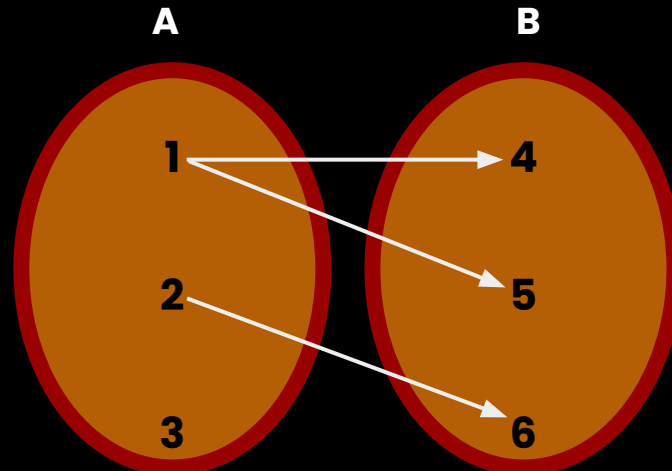
FUNCTION

- **Let A and B be two non-empty sets. Then a function 'f' from set A to set B is a rule or method or correspondence which associates elements of set A to elements of set B such that:**
 - i. All elements of set A are associated to elements in set B.**
 - ii. An element of set A is associated to a unique element in set B.**

Que. Identity which of them is a function from A to B ?



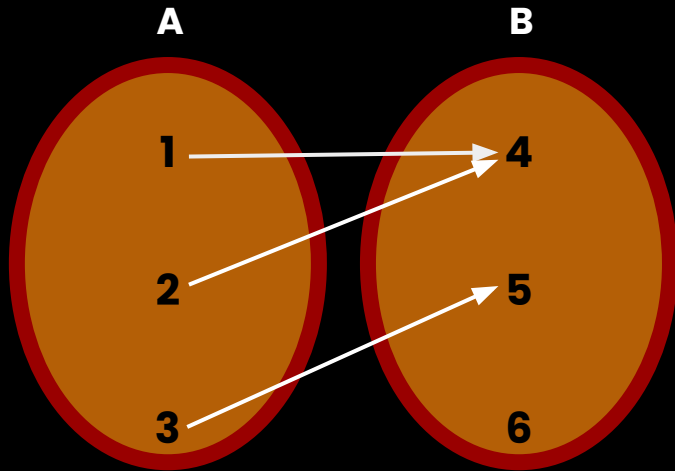
Not a Function



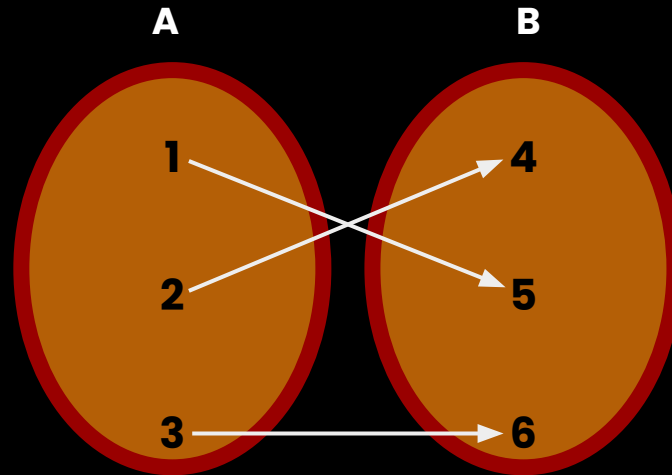
Not a Function

USE MY CODE : SS12

Que. Identity which of them is a function from A to B ?



Function



Function

USE MY CODE : SS12

June 2019

If $A = \{a, b, c, d\}$; $B = \{p, q, r, s\}$ which of the following relation is a function from A to B

(a) $R_1 = \{(a, p), (b, q), (c, s)\}$

(b) $R_2 = \{(p, a), (b, r), (d, s)\}$

(c) $R_3 = \{(b, p), (c, s), (b, r)\}$

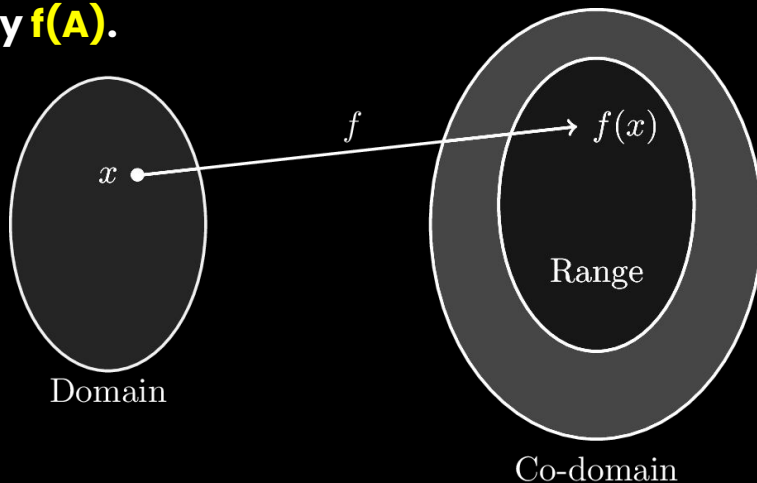
(d) $R_4 = \{(a, p), (b, r), (c, q), (d, s)\}$

Ans : d

USE MY CODE : SS12

Domain, Codomain and Range of Function

- If $f : A \rightarrow B$, the set A is known as the **domain** of f and the set B is known as the **co-domain** of f .
- The set of all f -images of elements of A is known as **the range of f or image set of A under f** and is denoted by **$f(A)$** .



Nov 2018

A is $\{1, 2, 3, 4\}$ and B is $\{1, 4, 9, 16, 25\}$ if a function f is defined from set A to B where $f(x) = x^2$ then the range of f is:

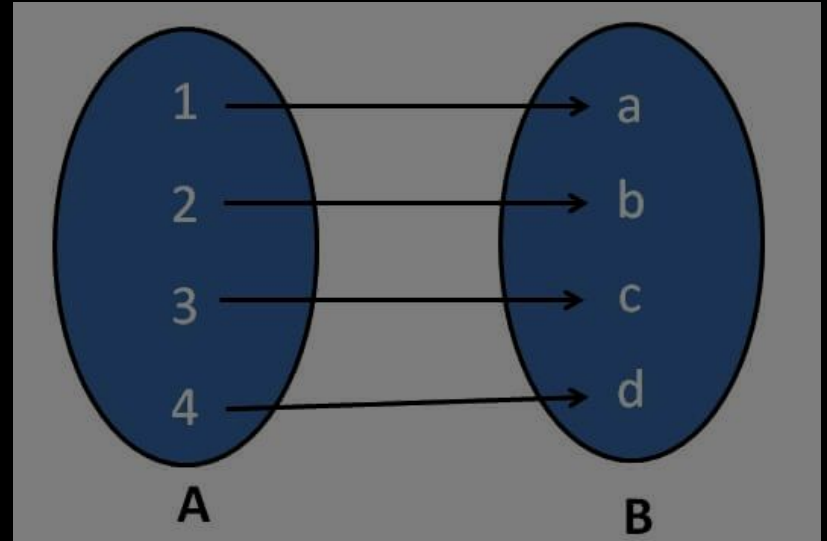
- (a) $\{1, 2, 3, 4\}$**
- (b) $\{1, 4, 9, 16\}$**
- (c) $\{1, 4, 9, 16, 25\}$**
- (d) None of these**

Ans: b

USE MY CODE : SS12

TYPES OF FUNCTION

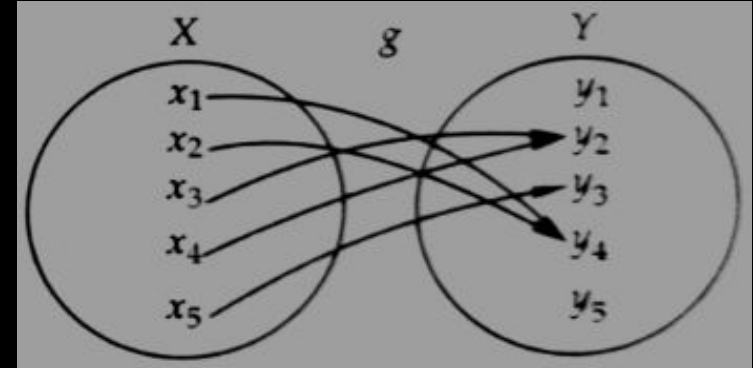
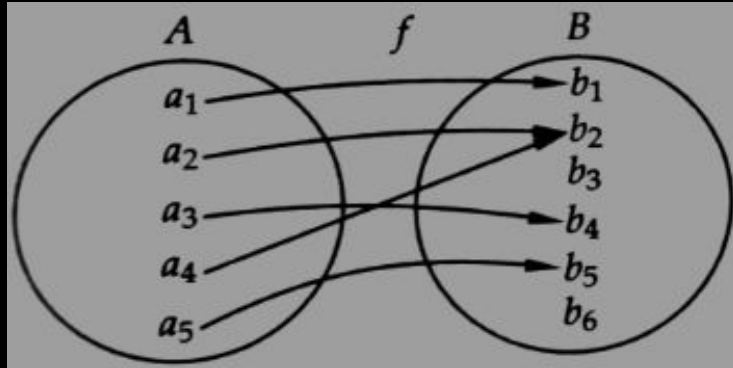
ONE - ONE FUNCTION



USE MY CODE : SS12

TYPES OF FUNCTION

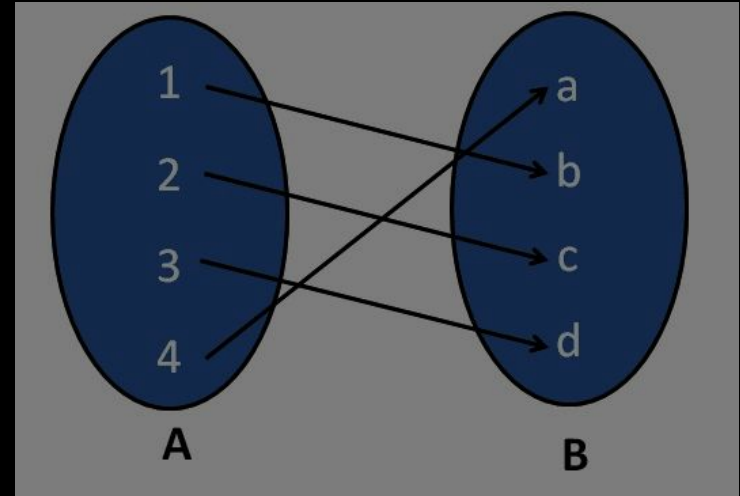
MANY ONE FUNCTION



USE MY CODE : SS12

TYPES OF FUNCTION

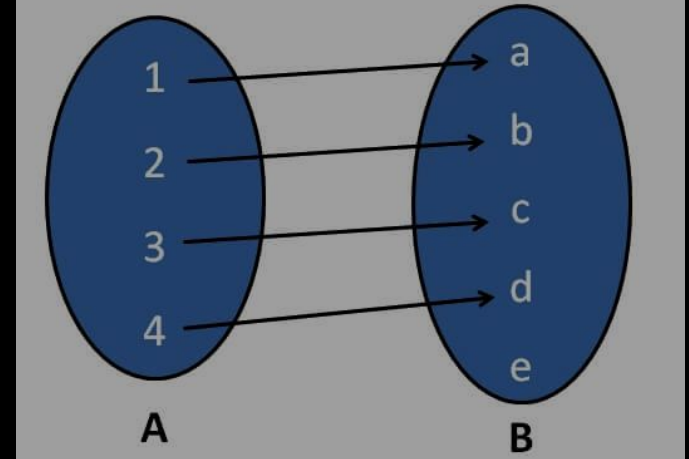
ONTO FUNCTION



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TYPES OF FUNCTION

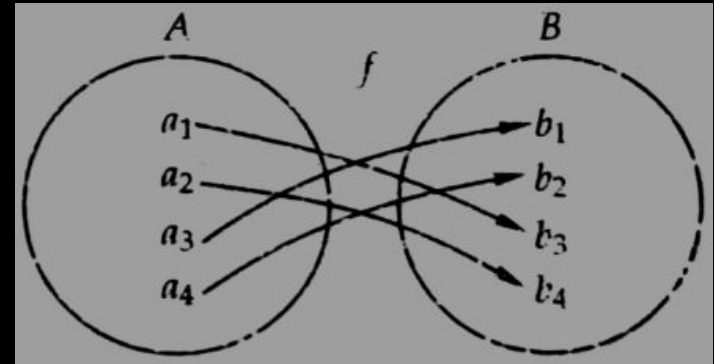
INTO FUNCTION



USE MY CODE : SS12

TYPES OF FUNCTION

BIJECTIVE FUNCTION (ONE ONE ONTO)



USE MY CODE : SS12

Dec 2014

**Let N be the set of all Natural number; E be the set of all even natural numbers
then the function**

$F : N \rightarrow E$ defined as $f(x) = 2x ; \forall x \in N$ is:

- (a) One-one into**
- (b) One-one onto**
- (c) Many-one into**
- (d) Many-one onto**

Ans : b

USE MY CODE : SS12

IDENTITY FUNCTION

- The function $f : \mathbb{R} \rightarrow \mathbb{R}$:

$$f(x) = x$$

- $\text{Dom}(f) = \mathbb{R}$ and $\text{Range}(f) = \mathbb{R}$

CONSTANT FUNCTION

$$f: R \rightarrow R:$$

$$f(x) = k$$

- $\text{Dom}(f) = R$ and
- $\text{Range}(f)$ is the singleton set $\{k\}$

Dec 2014

The No. of elements in range of constant function is

- (a) One**
- (b) Zero**
- (c) Infinite**
- (d) None**

Ans : a

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COMPOSITION OF FUNCTION

- $f \circ g(x) = f(g(x))$
- $g \circ f(x) = g(f(x))$

June 2019

If $f(x) = x^2$ and $g(x) = \sqrt{x}$ then

(a) $g \circ f(3) = 3$

(b) $g \circ f(-3) = 9$

(c) $g \circ f(9) = 3$

(d) $g \circ f(-9) = 3$

Ans : a

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July 2021

If $f(x) = x^2 - 1$ and $g(x) = |2x + 3|$, then $f \circ g(3) - g \circ f(-3) =$

(a) 71

(b) 61

(c) 41

(d) 51

Ans : b

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INVERSE OF FUNCTION

- Let $f : A \rightarrow B$ be one -one and onto function , then there exist a unique function $g : B \rightarrow A$,
such that $f(x) = y \Leftrightarrow g(y) = x \quad \forall x \in A , y \in B$.
. Then g is said to be inverse of f . Thus $g = f^{-1}$

INVERSE OF FUNCTION

• ALGORITHM

Let $f : A \rightarrow B$ be a bijection . To find the inverse of f we follow the following steps :

STEP 1: Put $f(x) = y$

STEP 2: Solve $f(x) = y$ to obtain x in terms of y

STEP 3: In the relation obtained in Step 2 replace x by $f^{-1}(y)$ to obtain the required inverse

Dec 2021 , June 2022

If $u(x) = 1/(1-x)$, then $u^{-1}(x)$ is

(a) $1/(x-1)$

(b) $1-x$

(c) $1-(1/x)$

(d) $(1/x)-1$

Ans : C

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EQUAL FUNCTION

- **Two function f and g are said to be equal , written as $f=g$ if they have the same domain and they satisfy the condition $f(x) = g(x)$, for all x**

Concept Of Limit

We say that $\lim_{x \rightarrow a} f(x) = l$ if whenever $x \rightarrow a$, $f(x) \rightarrow l$.

Concept Of Limit

- We consider a function $f(x) = 2x$.
- If x is a number approaching to the number 2 then $f(x)$ is a number approaching to the value $2(2) = 4$
- The following table shows $f(x)$ for different values of x approaching 2

x	f(x)
1.90	3.8
1.99	3.98
1.999	3.998
1.9999	3.9998

- *Here x approaches 2 from values of $x < 2$ and for x being very close to 2 ,f(x) is very close 4. This situation is defined as left-hand limit of f(x) as x approaches 2 and is written as $\lim_{x \rightarrow 2^-} f(x) = 4$*

x	f(x)
2.0001	4.0002
2.001	4.002
2.01	4.02

- Here x approaches 2 from values of x greater than 2 and for x being very close 2, $f(x)$ is very close to 4. This situation is defined as right-hand limit of $f(x)$ as x approaches 2 and is written as $\lim_{x \rightarrow 2^+} f(x) = 4$

Existence of a Limit

- $\lim_{x \rightarrow a} f(x)$ is said to exist when both left-hand limits and right hand limits exists and they are equal

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$$

LHL and RHL Concept

Example: $f(x) = \begin{cases} -3x & \text{when } x < 0 \\ 2x & \text{when } x > 0 \end{cases}$ s:

Test the existence of $\lim_{x \rightarrow 0} f(x)$.

Important Results

$$\lim_{x \rightarrow a} \{ f(x) + g(x) \} = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} \{ f(x) - g(x) \} = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} \{ f(x) \cdot g(x) \} = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

Important Results

$$\lim_{x \rightarrow a} \{ f(x) / g(x) \} = \left\{ \lim_{x \rightarrow a} f(x) \right\} / \left\{ \lim_{x \rightarrow a} g(x) \right\} :$$

$$\lim_{x \rightarrow a} c = c \text{ where } c \text{ is constant}$$

$$\lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x)$$

Methods Of Solving

- DIRECT SUBSTITUTION**

Rule 1 : Put $x = a$ in the given function. If $f(a)$ is a definite value then

$$\lim_{x \rightarrow a} f(x) = f(a).$$

Example:

$$\lim_{x \rightarrow 1} (x^2 + 5x - 2) = (1^2 + 5 \times 1 - 2) = 4.$$

Example 1: Evaluate (i) $\lim_{x \rightarrow 2} (3x + 9);$

INDETERMINATE FORMS OF LIMITS

$$\frac{\infty}{\infty}, \frac{0}{0}, \infty - \infty, 0^0, 0 \cdot \infty, \infty^0, 1^\infty$$

- If $f(a)$ is indeterminate, we adopt the rules given below.



Rule 2: If $f(x)$ is a rational function then factorize the numerator and the denominator. Cancel out the common factors and then $x = a$.

Example: Evaluate (i) $\lim_{x \rightarrow 3} \left(\frac{x^2 - 9}{x - 3} \right)$

- If $f(a)$ is indeterminate, we adopt the rules given below.



Rule 3: If the given function contains a surd then simplify it by using conjugate surds. After simplification, put $x = a$.

Example: Evaluate $\lim_{x \rightarrow 0} \left\{ \frac{\sqrt{1+x} - \sqrt{1-x}}{x} \right\}$

Some Important Limits

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$$

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \rightarrow a} \left(\frac{x^n - a^n}{x - a} \right) = na^{n-1},$$

Where $a > 0$

Some Important Limits

$$\lim_{x \rightarrow a} \left(\frac{x^n - a^n}{x - a} \right) = na^{n-1}, \quad \text{Where } a > 0$$

Evaluate $\lim_{x \rightarrow a} \left\{ \frac{x^{12} - a^{12}}{x - a} \right\}.$

Some Important Limits

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

Evaluate $\lim_{x \rightarrow 0} \left(\frac{e^{3x} - 1}{x} \right).$



$$\lim_{x \rightarrow a} f(x)^{g(x)} = e^{\lim_{x \rightarrow a} g(x) \{f(x) - 1\}}$$

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Example: Find $\lim_{x \rightarrow a} \left(1 + \frac{9}{x}\right)^x$. (Form 1)

TRICK : Limits Of Rational Functions

$\frac{\infty}{\infty}$

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$$

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TRICK : Limits Of Rational Functions

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)} = \frac{a_0 + a_1x + a_2x^2 + \dots + a_mx^m}{b_0 + b_1x + b_2x^2 + \dots + b_nx^n}$$

$$m = n$$

$$\frac{a_m}{b_n}$$

$$m < n$$

$$0$$

$$m > n$$

$$\infty$$

$$a_m b_n > 0$$

$$m > n$$

$$-\infty$$

$$a_m b_n < 0$$

Example: Find $\lim_{x \rightarrow \infty} \frac{2x+1}{x^3+1}$.

$\left[\text{Form } \frac{\infty}{\infty} \right]$

TRICK : L' HOSPITAL RULE

$$\frac{\infty}{\infty}, \frac{0}{0},$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\infty}{\infty}, \frac{0}{0},$$

Both $f(x)$ and $g(x)$ are continuous and differentiable at $x = a$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

Continue the process till you get a finite answer

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- **By the term continuous we mean some thing which goes on without interruption and without abrupt changes.**



CONTINUITY

- A function $f(x)$ is said to be continuous at $x = a$ if and only if
 - (i) $f(x)$ is defined $x = a$
 - (ii) $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$
 - (iii) $\lim_{x \rightarrow a} f(x) = f(a)$

CONTINUITY

- A function $f(x)$ is said to be continuous at $x = a$ if and only if

$$\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$$

OR

$$\lim_{x \rightarrow a} f(x) = f(a)$$



- i. The sum, difference and product of two continuous functions is a continuous function.**
- ii. The quotient of two continuous functions is continuous function provided the denominator is not equal to zero.**

Important results on continuous function

A constant function $f(x) = k$, is continuous everywhere.

Identity function $f(x) = x$, is continuous everywhere.

A polynomial function $f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n, n \in \mathbb{N}, x \in \mathbb{R}$, is continuous everywhere.

Important results on continuous function

The modulus function $f(x) = |x|$ is continuous everywhere.

The logarithmic function is continuous in $(0, \infty)$.

The exponential function is continuous everywhere.

Example $f(x) = \begin{cases} \frac{1}{2} - x & x < 1/2 \end{cases}$

$$= \frac{3}{2} - x \quad \text{when } \frac{1}{2} < x < 1$$

$$= \frac{1}{2} \quad \text{when } x = \frac{1}{2}$$

Discuss the continuity of $f(x)$ at $x = \frac{1}{2}$

Example Find points of discontinuity of the function $f(x) = \frac{x^2 + 2x + 5}{x^2 - 3x + 2}$